KARNATAKA ICSE SCHOOLS ASSOCIATION

ISC STD. XII Preparatory Examination 2026

Subject – MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 80** **Date: 12.01.2026**

(Candidates are allowed **additional 15 minutes** for **only** reading the paper.
They must **NOT** start writing during this time.)
This Question Paper consists of three sections A, B and C
Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C.**
Section A: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each**.
Section B: Internal choice has been provided in **one question of two marks and one question of four marks.**
Section C: Internal choice has been provided in **one question of two marks and one question of four marks.**
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION A (65 marks)

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed. $[15 \times 1 = 15]$

(i) If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|adj\ A|$

(a) 64
(b) 16
(c) 0
(d) 8

(ii) The order and degree of $y = x\frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$

(a) 1 and 2
(b) 2 and 1
(c) 2 and 2
(d) 1 and 1

(iii) If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$ then $\frac{dy}{dx}$ is

(a) $\frac{x-1}{y-1}$
(b) $\frac{x-1}{y+1}$
(c) $\frac{y-1}{x+1}$
(d) $\frac{y+1}{x-1}$

(iv) Assertion (A): If A and B are square matrices of order 3 such that $|A| = 2, |B| = -3$, then $|2AB| = -48$.
Reason (R): If A be any given square matrix of order n, then $A(adj\ A) = (adj\ A)A = |A|I$, where I is the identity matrix of order n.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

(v) For any given event A
Statement 1: Event A and null event $\emptyset$ are always independent.
Statement 2: Event A and sure event S are always independent.

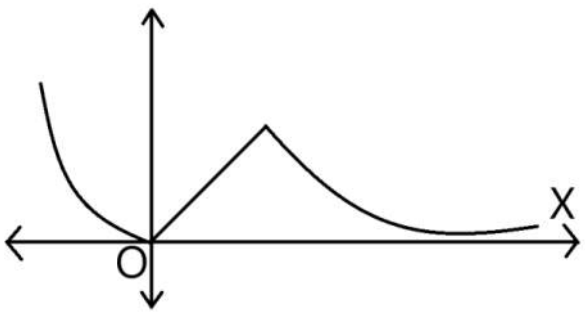
(a) Both statements are true.
(b) Both statements are false.
(c) Statement 1 is true and Statement 2 is false.
(d) Statement 1 is false and Statement 2 is true.

(vi) If A is square matrix such that $A^2 = A$, then $(I + A)^2 - 3A =$

(a) I
(b) 0
(c) $-I$
(d) A

(vii) Observe the graph of the given function and select the correct answer from the following

$$f(x) = \begin{cases} x^2, \ when\ x < 0 \\ x, \ when\ 0 \leq x < 1 \\ \frac{1}{x}, \ when\ 1 \leq x < \infty \end{cases}$$



(a) $f(x)$ is not continuous at $x = 0$ and $x = 1$.
(b) $f(x)$ is not continuous and not differentiable at $x = 0$ and $x = 1$.
(c) $f(x)$ is continuous at $x = 0$ and $x = 1$, but not differentiable.
(d) $f(x)$ is continuous and differentiable at $x = 0$ but not at $x = 1$.

(viii) Let $f: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\} : f(x) = \frac{4x}{3x+4}$. Then $f^{-1}(y) =$

(a) $\frac{4y}{4-3y}$
(b) $\frac{4y}{4y+3}$
(c) $\frac{4y}{3y-4}$
(d) $\frac{4y}{3y+4}$

(ix) A relation R is defined on a set $A = \{1, 2, 3\}$ given as
$R = \{(1,1), (2,2), (2,3), (3,2), (3,3)\}$.
Statement 1: R is an identity relation.
Statement 2: R is a reflexive relation but not identity relation.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is false, Statement 2 is true
(c) Both the statements are true.
(d) Both the statements are false.

(x) If $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$, then $x =$

(a) $\frac{1}{2}$
(b) $\frac{1}{4}$
(c) $\frac{1}{6}$
(d) $\frac{1}{8}$

(xi) Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$. What is the probability that only one of them is selected?

(xii) Find the value of k, if $A = \begin{bmatrix} 1 & k & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is not invertible.

(xiii) Let A = {0,1,2,3}. A relation R on A is defined as
R $\{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$. Is R transitive? Give reason.

(xiv) If set A contains 5 elements and set B contains 6 elements, then find the number of one-one and onto mappings from A to B. Explain.

(xv) A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Question 2 [2]

(i) If $y = \sqrt{e^{\sqrt{x}}}$, find $\frac{dy}{dx}$

OR

(ii) A balloon which always remain spherical has a variable diameter $\frac{2}{3}(3x + 1)$. Find the rate of change of its volume with respect to x.

Question 3 [2]

Find the interval on which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is increasing.

Question 4 [2]

Find the general solution of the differential equation $\frac{dy}{dx} = \sqrt{4 - y^2}\,; -2 < y < 2$

Question 5 [2]

(i) Evaluate : $\int e^x \left(\frac{2+sin2x}{\cos^2 x}\right) dx$

OR

(ii) Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan x \, dx$

Question 6 [2]

Consider the mapping $f: A \to B$ defined by $f(x) = \frac{x-1}{x-2}$ such that f is a bijection.

(i) Find the domain of f

(ii) Find f^{-1}(x)

Question 7 [4]

Prove that $\sin(2\tan^{-1}\frac{3}{5} - \sin^{-1}\frac{7}{25}) = \frac{304}{425}$

Question 8 [4]

If y = $(\sin^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{\mathrm{dx}} = 2$

Question 9 [4]

(i) Prove that $\int_0^{\pi} \frac{xtanx}{secx+tanx} dx = \pi(\frac{\pi}{2} - 1)$

OR

(ii) Solve the differential equation $(x\sqrt{x^2 + y^2} - y^2)dx + xydy = 0$.

Question 10 [4]

(i) In a kabaddi league, two matches are being played between Jaipur and Delhi. It is assumed that the outcomes of two games are independent. The probability of Jaipur winning, drawing and losing the game against Delhi are $\frac{1}{2}$, $\frac{3}{10}$ and $\frac{1}{5}$ respectively. Each team gets 5 points for win, 3 points for draw and 0 points for loss in a game. After two games, find the probability that

a) Jaipur has more points than Delhi

b) Jaipur and Delhi have equal points

OR

(ii) In a class of 75 students, 15 are above average, 45 are average and the rest are below average achievers. The probability that an above average achieving student fails is 0.005, that an average achieving student fails is 0.05 and that of a below average achieving student is 0.15. If a student is known to have passed, what is the probability that he is a below average achiever?

Question 11 [6]

Rajesh wants to purchase some fruits from fruit market. 4kg apples, 3kg grapes and 2kg oranges cost him ₹ 600, 2kg apples, 4kg grapes and 6kg oranges cost him ₹ 900 and 6kg apples, 2kg grapes and 3kg oranges cost him ₹ 700.

(i) Express the given data in the form of a set of simultaneous equations.

(ii) Solve the system of equations formed by Martin's method.

(iii) Find how much Rajesh has to pay per kg for each fruit.

Question 12 [6]

(i) Evaluate: $\int \frac{1}{1+x^4}\,dx$

OR

(ii) Evaluate: $\int (3x-2)\sqrt{x^2+x+1}\,dx$

Question 13 [6]

(i) A farmer wants to construct a square tank of capacity 250 cubic metres that has to be dug out from the field.

(a) If h is height of the tank to be dug with a square base of x, then find relation between the variables.

(b) If the cost of the land is ₹ 50 per square metre and cost of digging increases with the depth and for the whole tank cost is ₹ $400h^2$, then determine cost C.

(c) If the builder wants to minimize the cost of the tank, then find depth.

(d) How much money will builder spend to construct this tank?

OR

(ii) A rectangular window is surrounded by an equilateral triangle. Given that the perimeter is 16m, find the width of the window so that the maximum amount of light may enter.

Question 14 [6]

Three defective bulbs are mixed with 7 good ones. Let X be the number of defective bulbs when 3 bulbs are drawn at random. Find the probability distribution of X and calculate its mean.

Section B (15 marks)

Question 15

In sub parts (i) to (iii) choose the correct option and answer the other subparts as instructed. [5 x 1=5]

(i) Assertion (A): If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$, then $\vec{a}.\vec{b} = 7$.

Reason (R): Lagrange's identity is $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2$.

a. Both A and R are true, R is the correct explanation of A
b. Both A and R are true, R is not the correct explanation of A
c. A is true and R is false.
d. A is false and R is true.

(ii) A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{\imath} + 15\hat{\jmath} + 6\hat{k}$ and the other is along the vector $2\hat{\imath} + 10\hat{\jmath} + \lambda\hat{k}$, then the value of λ is

a. 6
b. 1
c. $\frac{1}{4}$
d. 4

(iii) A straight line drawn through the points $(-6, -6, 5)$ and $(12, -6, 1)$ meets xy-plane at

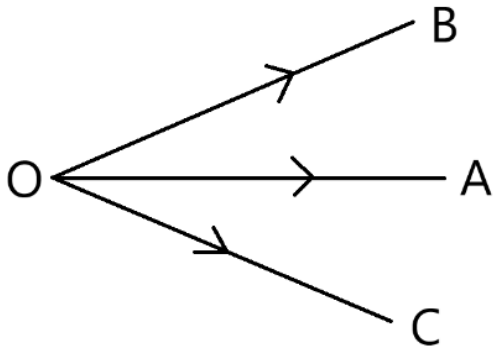
a. $(9, -4, 0)$
b. $(-9, -4, 0)$
c. $(-9, 4, 0)$
d. $(9, 4, 0)$

(iv) Find the angle between the unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a} - \vec{b}$ is also a unit vector.

(v) The vectors from origin to the point A and B are $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ respectively. Find the area of ΔOAB.

Question 16 [2]

(i) Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his pre-decided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$.



i. Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

ii. If $\vec{a}.\vec{b} = 1$, distance of O to A is 1km and that from O to B is 2km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

OR

(ii) If $\vec{a}$ and $\vec{b}$ are two-unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$, then find the value of $\left(2\vec{a} - 5\vec{b}\right).(3\vec{a} + \vec{b})$

Question 17 [4]

(i) Find the shortest distance between the pair of lines whose equations are $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$.

OR

(ii) From the point $P(1,2,4)$, a perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the length and co-ordinates of the foot of the perpendicular.

Question 18 [4]

Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

SECTION C

Question 19

In sub parts (i) to (iii) choose the correct option and answer the other subparts as instructed. [5 x 1=5]

(i) Assertion (A): If $C(x) = 20000 + \frac{21}{x}$ and $R(x) = 6x$ then profit function is $P(x) = \frac{29x}{10} - 20000$.

Reason (R): Profit function is $P(x) = R(x) - C(x)$

a. Both A and R are true, R is the correct explanation of A.
b. Both A and R are true, R is not the correct explanation of A
c. A is true and R is false.

d. A is false and R is true.

(ii) Rohit joins a career counselling institute as a counsellor. The manager says, "Within a year 1 want a breakeven point. For that I will give you ₹ 24,000 fixed salary per month and the variable salary will be 25% of the revenue recovered on hiring students at the rate of ₹ 800 charged per student". Find how many students should be admitted by Rohit in a year in the institute to fulfil his manager's condition

a. 30
b. 40
c. 80
d. 100

(iii) If two regression lines are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$ then $b_{xy} + b_{yx}$ is

a. $\frac{3}{4}$
b. $-\frac{3}{4}$
c. $-\frac{3}{2}$
d. $\frac{3}{2}$

(iv) Find $\bar{x}$ and $\bar{y}$ if the two regression lines are $x + 2y = 5$ and $2x + 3y = 8$.

(v) If the average revenue function is $9 - x^2$, find marginal revenue function.

Question 20 [2]

(i) Anuj and Ashish launched a new fountain pen in their pen-factory which is consisting of ₹ 6400 as overheads, ₹ 35 per pen as the cost of material and labour cost ₹ $\frac{x^2}{100}$ for x items produced. Find the values of x for which average cost is increasing.

OR

(ii) A monopolist's demand function is $x = 60 - \frac{P}{5}$. Find the total revenue and marginal revenue function. At what level of output is marginal revenue zero.

Question 21 [4]

From the given data with correlation coefficient $\frac{2}{3}$

Variable	x	y
Mean	6	8
Variance	16	36

Find (i) regression coefficients b_{yx} and b_{xy}
(ii) regression line of x on y
(iii) most likely value of x when $y = 14$.

Question 22 [4]

(i) An aeroplane of an airline can carry a maximum of 200 passengers. A profit of ₹ 400 is made on each first class and a profit of ₹ 300 is made on each economy class ticket. The airline reserves atleast 20 seats for first class. However, atleast 4 times as many passengers prefer to travel by economy class than by first class. Determine how many

of each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit? Solve graphically.

OR

(ii) A company produces soft drinks that has a contract which requires a minimum of 80 units of the chemical A and 60 units of the chemical B to go into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier S has a mix of 4 units of A and 2 units of B that costs ₹ 10. The supplier T has a mix of 1 unit of A and 1 unit of B that costs ₹ 4. How many mixes from S and T should the company purchase to honour contract requirement and yet minimize cost?

KARNATAKA ICSE SCHOOLS ASSOCIATION

ISC STD. XII Preparatory Examination 2026

Subject – Mathematics

MARKING SCHEME

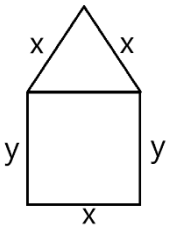
Q.No.	Answer	Approx. mark allocation
Q1 (i)	(a)	1
(ii)	(a)	1
(iii)	(c)	1
(iv)	(b)	1
(v)	(a)	1
(vi)	(a)	1
(vii)	(c)	1
(viii)	(a)	1
(ix)	(b)	1
(x)	(a)	1
(xi)	A→ Amit is selected B→ Nisha is selected $P(A) = \frac{1}{5}$ $P(B) = \frac{1}{6}$ $P(\text{only one selected}) = P(A\bar{B}) + P(\bar{A}B)$ $= \frac{1}{5} \times \frac{5}{6} + \frac{4}{5} \times \frac{1}{6}$ $= \frac{9}{30} = \frac{3}{10}$	1
(xii)	If $\lvert A\rvert = 0$, it is not invertible $\begin{vmatrix} 1 & k & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{vmatrix} = 1(2-5) - k(1-10) + 2(1-4) = 0$ $= -3 + 9k - 6 = 0$ $k = 1$	1
(xiii)	$(1,0), (0,3) \in R$ but $(1,3) \notin R$ Therefore, R is not transitive	1
(xiv)	Number of one-one and onto functions from A to B = 0 as the number of elements in A and B are not equal.	1
(xv)	There are 8 even and 9 odd numbers from 1 to 17 $P(both\ even) = \frac{^8C_2}{^{17}C_2} = \frac{7}{34}$	1
2. (i)	$y = \sqrt{e^{\sqrt{x}}}$ $y^2 = e^{\sqrt{x}}$ Differentiate with respect to x, $2y\frac{dy}{dx} = e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}$ $\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.\frac{e^{\sqrt{x}}}{2\sqrt{x}} = \frac{1}{4\sqrt{x}}.\sqrt{e^{\sqrt{x}}}$	1 1
(ii)	Diameter $= \frac{2}{3}(3x+1)$ $r = \frac{1}{3}(3x+1)$ Volume $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dx} = \frac{dV}{dr} \times \frac{dr}{dx} = \frac{4}{3} \times 3\pi r^2 \times \frac{1}{3} \times 3$ $= 4\pi \frac{(3x+1)^2}{9}$	1 1

3.	$f(x) = -2x^3 - 9x^2 - 12x + 1$ $f'(x) = -6x^2 - 18x - 12 \geq 0$ when increasing $-6(x^2 + 3x + 2) \geq 0$ $x^2 + 3x + 2 \leq 0$ $(x + 2)(x + 1) \leq 0 \qquad x \in [-2, -1]$ $f(x)$ is increasing on the interval $[-2, -1]$	1 1
4.	$\frac{dy}{dx} = \sqrt{4 - y^2} \qquad \int \frac{dy}{\sqrt{4-y^2}} = \int dx$ $\sin^{-1}\left(\frac{y}{2}\right) = x + c$ is the required equation.	1 1
5. (i)	$\int e^x \left(\frac{2+\sin 2x}{\cos^2 x}\right) dx = \int e^x \left(2\sec^2 x + \frac{2\sin x \cos x}{\cos^2 x}\right) dx$ $= 2\int (e^x \sec^2 x + e^x \tan x)\, dx \qquad \int e^x[f(x) + f'(x)]dx$ $= 2e^x \tan x + c \qquad = e^x f(x) + c$	1 1
(ii)	Let $I = \int_0^{\frac{\pi}{2}} \sin 2x \log \tan x \, dx$ … (1) $\qquad \int_0^a f(x)dx = \int_0^a f(a - x)\, dx$ Also $I = \int_0^{\frac{\pi}{2}} \sin 2\left(\frac{\pi}{2} - x\right) \log \tan\left(\frac{\pi}{2} - x\right) dx$ $I = \int_0^{\frac{\pi}{2}} \sin 2x \log \cot x \, dx$ … (2) Adding (1) and (2), $2I = \int_0^{\frac{\pi}{2}} \sin 2x \log(\tan x \,.\, \cot x)\, dx$ $2I = \int_0^{\frac{\pi}{2}} \sin 2x \log 1 \, dx$ $2I = 0$ Therefore, $\int_0^{\frac{\pi}{2}} \sin 2x \log \tan 2x \, dx = 0$	1 1
6.	$f(x) = \frac{x-1}{x-2}$ Domain of f *is* $R - \{2\}$ Let $y = \frac{x-1}{x-2} \rightarrow xy - 2y = x - 1$ $x = \frac{2y-1}{y-1}$ $f^{-1}(x) = \frac{2x-1}{x-1}$	1 1
7.	$\sin^{-1}\left(\frac{7}{25}\right) = \tan^{-1}\left(\frac{7}{24}\right)$ $2tan^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{2 \cdot \frac{3}{5}}{1 - \frac{9}{25}}\right) = \tan^{-1}\left(\frac{6}{5} \cdot \frac{25}{16}\right)$ $= \tan^{-1}(\frac{15}{8})$ $\sin\left(2\tan^{-1}\frac{3}{5} - \sin^{-1}\frac{7}{25}\right) = \sin(\tan^{-1}\frac{15}{8} - \tan^{-1}\frac{7}{24})$ $= \sin\left(\tan^{-1}\left(\frac{\frac{15}{8} - \frac{7}{24}}{1 + \frac{15}{8} \cdot \frac{7}{24}}\right)\right)$ $= \sin\left(\tan^{-1}\left(\frac{\frac{304}{192}}{\frac{192+105}{192}}\right)\right) = \sin\tan^{-1}(\frac{304}{297})$ $= \sin\sin^{-1}(\frac{304}{425}) = \frac{304}{425}$	1 1 1 1

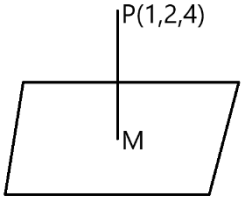
8.	$y = (\sin^{-1} x)^2$ $\frac{dy}{dx} = 2\sin^{-1} x \ \frac{1}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x$ Diff. w.r.t.x, $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\ \frac{-2x}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$ $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$	1 LHS (1) RHS (1) 1
9. (i)	$I = \int_0^\pi \left(\frac{x\,tanx}{\sec x+\tan x}dx\right)$ --- (1) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ Also $I = \int_0^\pi \left(\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\right)dx$ --- (2) (1) + (2) $2I = \pi\int_0^\pi \left(\frac{tanx}{\sec x+\tan x}\right)dx$ $I = \frac{\pi}{2}\int_0^\pi \left(\frac{\tan x(\sec x-tanx)}{(\sec x+\tan x)\,(secx-tanx)}\right)dx$ $= \frac{\pi}{2}\int_0^\pi (tanxsecx - \tan^2 x)dx$ $\sec^2 x - \tan^2 x = 1$ $= \frac{\pi}{2}\int_0^\pi (tanxsecx - \sec^2 x + 1)dx$ $= \frac{\pi}{2}\,[\,secx - tanx + x\,]_0^\pi$ $= \frac{\pi}{2}\,[-1-0+\pi] - \frac{\pi}{2}[1-0+0]$ $= \frac{\pi}{2}\times -2 + \frac{\pi}{2}\pi = \pi(\frac{\pi}{2}-1)$	1 1 1 1
(ii)	$\left(x\sqrt{x^2+y^2} - y^2\right)dx + xydy = 0$ $\frac{dy}{dx} = \frac{y^2 - x\sqrt{x^2+y^2}}{xy}$ $= \frac{\left(\frac{y}{x}\right)^2 - \sqrt{1+\left(\frac{y}{x}\right)^2}}{\frac{y}{x}}$ Let y = vx, $\frac{dy}{dx} = V + x\frac{dv}{dx}$ $V + x\frac{dv}{dx} = \frac{V^2 - \sqrt{1+V^2}}{V}$ = V - $\frac{\sqrt{1+V^2}}{V}$ $= x\frac{dv}{dx} = \frac{-\sqrt{1+V^2}}{V}$ $\int \frac{V}{\sqrt{1+V^2}}\,dv = -\int \frac{dx}{x}$ $\sqrt{1+V^2} = -log\lvert x\rvert + C$ $\sqrt{x^2+y^2}$ + x log |x| = Cx	1 1 1 1
10. (i)	P(winning of Delhi) = P(losing of Jaipur) = $\frac{1}{5}$ P (losing of Delhi) = P (winning of Jaipur = $\frac{1}{2}$	

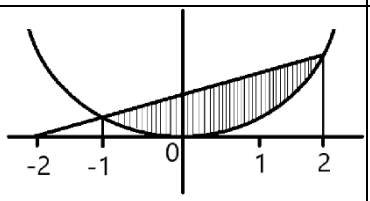
	P (drawing of Delhi) = P (drawing of Jaipur) = $\frac{3}{10}$ Let X be points of Jaipur and Y that of Delhi a) P (X > Y) = P(Jaipur win in first and second) + P(Jaipur win in first and draw in second) + P(Jaipur draw in first and win in second) $=\frac{1}{2}.\frac{1}{2}+\frac{1}{2}.\frac{3}{10}+\frac{3}{10}.\frac{1}{2}=\frac{11}{20}$ b) P(X = Y) = P(Jaipur win in first) . P(Delhi win in second) +P(Delhi win in first) . P(Jaipur win in second) + P(Jaipur and Delhi draw in first and second) $=\frac{1}{2}.\frac{1}{5}+\frac{1}{5}.\frac{1}{2}+\frac{3}{10}.\frac{3}{10}=\frac{29}{100}$	1 1 1 1
(ii)	P (student is above average achiever) = P(E_1) = $\frac{15}{75}=\frac{1}{5}$ P(student is average achiever) = P(E_2) = $\frac{45}{75}=\frac{3}{5}$ P(student is below average achiever = P(E_3) = $\frac{1}{5}$ A: Event that a student is known to have passed P(A/E_1) = 1-0.005 = 0.995 P(A/E_2) = 1-0.05 = 0.95 P(A/E_3) = 1-0.15 = 0.85 P(E_3/A) = $\frac{P(E_3).P(A/E_3)}{P(E_1).P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$ $=\frac{\frac{1}{5}\times 0.85}{\frac{1}{5}\times 0.995+\frac{3}{5}\times 0.95+\frac{1}{5}\times 0.85}$ = 0.18	1 1 1 1
11.	Let x be price of apples, y that of grapes and z that of oranges per kg. 4x + 3y + 2z = 600 2x + 4y + 6z = 900 6x + 2y + 3z = 700 **AX = B** $\begin{bmatrix}4 & 3 & 2\\2 & 4 & 6\\6 & 2 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}600\\900\\700\end{bmatrix}$ $\|A\|$ = 4 (12 – 12) – 3 (6-36)+2 (4-24) = 50 Adj A = $\begin{bmatrix}0 & -5 & 10\\30 & 0 & -20\\-20 & 10 & 10\end{bmatrix}$ $X = A^{-1}B = \frac{(adjA)\,B}{\|A\|}$ $=\frac{1}{50}\begin{bmatrix}0 & -5 & 10\\30 & 0 & -20\\-20 & 10 & 10\end{bmatrix}\begin{bmatrix}600\\900\\700\end{bmatrix}$ $=\frac{1}{50}\begin{bmatrix}2500\\4000\\4000\end{bmatrix}=\begin{bmatrix}50\\80\\80\end{bmatrix}$ Cost of apples = ₹ 50, Grapes = ₹ 80, Oranges = ₹ 80	1 1 1 1 1 1
12(i)	$I=\int\frac{dx}{1+x^4}=\frac{1}{2}\int\frac{2dx}{x^4+1}=\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{x^4+1}dx$ $=\frac{1}{2}\int\frac{x^2+1}{x^4+1}dx-\frac{1}{2}\int\frac{x^2-1}{x^4+1}dx$	1

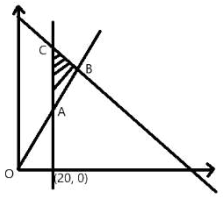
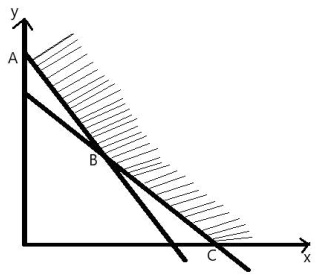
	I $= \frac{1}{2}(I_1 - I_2)$ $I_1 = \int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$ Put $x - \frac{1}{x} = t$ $(1+\frac{1}{x^2})dx = dt$ $= \int \frac{dt}{t^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}(\frac{t}{\sqrt{2}}) + c$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) + c$ $I_2 = \int \frac{x^2-1}{x^4+1}dx = \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$ Put $x + \frac{1}{x} = u$ $(1-\frac{1}{x^2})dx = du$ $= \int \frac{du}{u^2-(\sqrt{2})^2} = \frac{1}{2\sqrt{2}}log\left\|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right\| + c$ $= \frac{1}{2\sqrt{2}}log\left\|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right\| + c$ $I = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) - \frac{1}{4\sqrt{2}}log\left\|\frac{x+1-\sqrt{2}x}{x+1+\sqrt{2}x}\right\| + c$	substitution 1 1 substitution 1 1 1
(ii)	I $= \int 3x-2)\sqrt{x^2+x+1}\,dx$ $3x-2 = A\frac{d}{dx}(x^2+x+1) + B$ $3x-2 = A\,(2x+1) + B$ $A = \frac{3}{2}$, $B = -\frac{7}{2}$ $I = \int \frac{3}{2}(2x+1)\sqrt{x^2+x+1}\,dx - \frac{7}{2}\int \sqrt{x^2+x+1}\,dx$ $\frac{3}{2}I_1 - \frac{7}{2}I_2$ Let $x^2+x+1 = t^2$ $(2x+1)dx = 2tdt$ $I_1 = \int 2t.t\,dt = \frac{2t^3}{3} + c = \frac{2}{3}(x^2+x+1)^{\frac{3}{2}} + c$ $I_2 = \int \sqrt{x^2+x+1}\,dx = \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dt$ $= \frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left\|x+\frac{1}{2}+\sqrt{x^2+x+1}\right\| + c$ $I = (x^2+x+1)^{\frac{3}{2}} - \frac{7(2x+1)\sqrt{x^2+x+1}}{2} - \frac{21}{16}\log\left\|\frac{2x+1}{2}+\sqrt{x^2+x+1}\right\| + C$	1 1 1 1 1 1 1
13.(i)	$V = 250 = x^2h$ $x^2 = \frac{250}{h}$ $C = 50x^2 + 400h^2$ $= \frac{12500}{h} + 400h^2$	1 1

	$\frac{dC}{dh} = -\frac{12500}{h^2} + 800h$ $\frac{dC}{dh} = 0\ ,\quad h = 2.5m$ $\frac{d^2C}{dh^2} = \frac{25000}{h^3} + 800$ When $h = 2.5m$, $\frac{d^2C}{dh^2} = 2400 > 0$ Therefore, Minimum cost when $h = 2.5m$ Money spent $= \frac{12500}{2.5} + 400 \times 2.5^2$ $= Rs.7500$	1 1 1 1
(ii)	Perimeter $= 3x + 2y = 16$ Area $= \frac{\sqrt{3}x^2}{4} + xy$ $= \frac{\sqrt{3}x^2}{4} + \frac{x(16-3x)}{2} = \frac{\sqrt{3}x^2}{4} + 8x - \frac{3x^2}{2}$ $\frac{dA}{dx} = \frac{\sqrt{3}x}{2} + 8 - 3x$ $\frac{dA}{dx} = 0\ ,\quad x\left(3 - \frac{\sqrt{3}}{2}\right) = 8$ $x = \frac{16}{6-\sqrt{3}}$ $\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2} - 3 = -2.134 < 0$ Maximum area when $x = \frac{16}{6-\sqrt{3}} = 3.75m$ 	1 1 1 1 1 1
14.	X can take values $0, 1, 2, 3$ $P(X = 0) = \frac{7C_3}{10C_3} = \frac{7}{24}$ $P(X = 1) = \frac{3C_1 \times 7C_2}{10C_3} = \frac{21}{40}$ $P(X = 2) = \frac{3C_2 \times 7C_1}{10C_3} = \frac{7}{40}$ $P(X = 3) = \frac{3C_3}{10C_3} = \frac{1}{120}$ Probability distribution is Table px values Mean $\mu = \Sigma p_i x_i = \frac{9}{10}$	1 2 1 1 1
	Section B	
15.(i)	(a)	1
(ii)	(d)	1
(iii)	(a)	1
(iv)	$\left\|\sqrt{3}\vec{a} - \vec{b}\right\| = 1$ $\left(\sqrt{3}\vec{a} - \vec{b}\right)^2 = 3\|\vec{a}\|^2 + \left\|\vec{b}\right\|^2 - 2\sqrt{3}\vec{a}.\vec{b} = 1$ $3 + 1 - 2\sqrt{3}\vec{a}.\vec{b} = 1$	

x	$P(x)$	px
0	$\frac{7}{24}$	0
1	$\frac{21}{40}$	$\frac{21}{40}$
2	$\frac{7}{40}$	$\frac{14}{40}$
3	$\frac{1}{120}$	$\frac{3}{120}$

	$3 = 2\sqrt{3}\vec{a}.\vec{b}$ $\qquad \vec{a}.\vec{b} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\qquad \theta = \frac{\pi}{6}$	1
(v)	Area of triangle $= \frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$ $= \frac{1}{2}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$ $= \frac{1}{2}\lvert -9\hat{\imath} + 2\hat{\jmath} + 12\hat{k}\rvert = \frac{1}{2}\sqrt{229}$ sq. units	1
16.(i) (a)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 5\vec{a} - 2\vec{b} - \vec{a}$ $= 4\vec{a} - 2\vec{b}$ $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 5\vec{a} - 2\vec{b} - \vec{b} = 5\vec{a} - 3\vec{b}$	1
(b)	$\vec{a}.\vec{b} = 1$ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = 1$ $1 \times 2\cos\theta = 1$ $\qquad \cos\theta = \frac{1}{2}$ $\qquad \theta = \frac{\pi}{3}$	1
(ii)	$\lvert\vec{a} + \vec{b}\rvert = \sqrt{3}$ Squaring $\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b} = 3$ $1 + 1 + 2\vec{a}.\vec{b} = 3$ $\vec{a}.\vec{b} = \frac{1}{2}$ $(2\vec{a} - 5\vec{b}).(3\vec{a} + \vec{b}) = 6\lvert\vec{a}\rvert^2 + 2\vec{a}.\vec{b} - 15\vec{b}.\vec{a} - 5\lvert\vec{b}\rvert^2$ $= 6 - 13\vec{a}.\vec{b} - 5$ $= 1 - 13 \times \frac{1}{2} = -\frac{11}{2}$	1 1
17(i)	The two lines are $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$ S.D. $= \left\lvert\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix} = 8\hat{\imath} + 8\hat{\jmath} + 4\hat{k}$ $\lvert\vec{b}_1 \times \vec{b}_2\rvert = \sqrt{64 + 64 + 16} = \sqrt{144} = 12$ $\vec{a}_2 - \vec{a}_1 = -4\hat{\imath} - \hat{k} - (6\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ $= -10\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ S.D. $= \left\lvert\frac{(-10\hat{\imath} - 2\hat{\jmath} - 3\hat{k}).(8\hat{\imath} + 8\hat{\jmath} + 4\hat{k})}{12}\right\rvert$ $= \left\lvert\frac{-80 - 16 - 12}{12}\right\rvert = \left\lvert-\frac{108}{12}\right\rvert = 9$ units	1 1 1 1
(ii)	Let M be the foot of the perpendicular from P. Equation of PM through $P(1,2,4)$ is $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = r$ Point M on the line is $(2r + 1, r + 2, -2r + 4)$ M lies on $2x + y - 2z + 3 = 0$ 	1

	i.e. $(2(2r+1)+(r+2)-2(-2r+4)+3=0$ $4r+2+r+2+4r-8+3=0$ $9r-1=0 \qquad r=\frac{1}{9}$ Foot of the perpendicular M is $\left(\frac{2}{9}+1,\ \frac{1}{9}+2,\ -\frac{2}{9}+4\right)$ $M\left(\frac{11}{9},\frac{19}{9},\frac{34}{9}\right)$ Distance $PM=\sqrt{\left(\frac{11}{9}-1\right)^2+\left(\frac{19}{9}-2\right)^2+\left(\frac{34}{9}-4\right)^2}$ $=\frac{1}{3}$ units	1 1 1
18.	Curve is $x^2=4y$ and line is $x=4y-2$ $x^2=x+2, \quad x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2,-1$  Required area $=\int_{-1}^{2}(y\ of\ line)dx-\int_{-1}^{2}(y\ of\ curve)\,dx$ $=\int_{-1}^{2}\left(\frac{x+2}{4}\right)dx-\int_{-1}^{2}\left(\frac{x^2}{4}\right)dx$ $=\frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2}-\left[\frac{x^3}{12}\right]_{-1}^{2}$ $=\frac{1}{4}\left[2+4-\frac{1}{2}+2\right]-\left[\frac{8+1}{12}\right]$ $=\frac{9}{8}=1\frac{1}{8}$ sq. units.	1 1 1 1
	Section C	
19.(i)	(d)	1
(ii)	(b)	1
(iii)	(c)	1
(iv)	$\bar{x}=1, \bar{y}=2$	1
(v)	AR $=9-x^2 \qquad R(x)=9x-x^3$ MR $=\frac{dR}{dx}=9-3x^2$	1
20(i)	$C(x)=6400+35x+\frac{x^2}{100}$ $AC=\frac{6400}{x}+35+\frac{x}{100}$ AC increases when $\frac{d}{dx}AC\geq 0$ i.e., $-\frac{6400}{x^2}+\frac{1}{100}\geq 0$ $\frac{x^2-640000}{100x^2}\geq 0 \qquad x\geq 800$	1 1
(ii)	$R(x)=px=(300-5x)x$ $=300x-5x^2$ $MR=300-10x$ $MR=0$ when $x=30$	1 1
21.(i)	$b_{yx}=r\frac{\sigma_y}{\sigma_x}=\frac{2}{3}\times\frac{6}{4}=1$ $b_{xy}=\mathrm{r}\frac{\sigma_x}{\sigma_y}=\frac{2}{3}\times\frac{4}{6}=\frac{4}{9}$	1 1
(ii)	Regression line of x on y is $x-\bar{x}=b_{xy}(y-\bar{y})$ $x-6=\frac{4}{9}(y-8)$ $9x-54=4y-32$	

	$9x - 4y = 22$	1
(iii)	When $y = 14,\ x = 8.67$	1
22.(i)	Let x tickets of first class and y tickets of economy class be sold. $x \geq 20,\quad y \geq 4x,\quad x+y \leq 200$ Profit function Z = 400x + 300y Corner points: A (20, 80) Z = 32000; B (40, 160) Z = 64000; C (20, 180) Z = 62000  Maximum profit is Rs. 64000 when X = 40 and y = 160	1 1 Graph 1 1
(ii)	Let x be the number of mixes of S and y be that of T the company should purchase . $4x + y \geq 80$ $2x + y \geq 60.$ $x \geq 0, y \geq 0$ Cost function C=10x + 4y Corner points: A (0, 80) C = 320; B (10, 40) C = 260; C (30, 0) C = 300  Minimum cost of Rs. 260 when x = 10 and y = 40	1 1 1 1